

A μ -ORDINARY HASSE INVARIANT

WUSHI GOLDRING, MARC-HUBERT NICOLE

ABSTRACT. We construct a generalization of the Hasse invariant for certain unitary Shimura varieties of PEL type whose vanishing locus is the complement of the so-called μ -ordinary locus. We show that the μ -ordinary locus of those varieties is affine. As an application, we strengthen a special case of a theorem of one of us (W.G.) on the association of Galois representations to automorphic representations of unitary groups whose archimedean component is a holomorphic limit of discrete series.

1. INTRODUCTION

Starting with the cornerstone work of Deligne-Serre [DS74] on classical weight one modular forms, the Hasse invariant has been a fundamental tool for constructing congruences between automorphic forms. In turn, the congruences that arise from the Hasse invariant have been used to construct automorphic Galois representations that do not appear directly in the étale cohomology of Shimura varieties ([Tay91], [Gol12]). One limitation of the Hasse invariant is that there exist many Shimura varieties for which the Hasse invariant is identically zero. This happens precisely for those Shimura varieties whose ordinary locus is empty.

The μ -ordinary locus introduced by Rapoport and Richartz [RR96] can be viewed as a substitute to the ordinary locus when the latter is empty. Indeed a theorem of Wedhorn states that, for a prime of good reduction and hyperspecial level, the μ -ordinary locus is dense [Wed99, Th.1.6.2]. It is therefore natural to seek a generalization of the Hasse invariant whose vanishing locus is the complement of the μ -ordinary locus. We construct such an invariant for Shimura varieties $\mathrm{Sh}(\mathbf{G}, \mathbf{X})$ of PEL-type such that $\mathbf{G}(\mathbf{R})$ is isomorphic to a unitary similitude group $\mathrm{GU}(a, b)$ for some positive integers a, b . This class includes Picard modular varieties.

1.1. Main Result. Suppose $\mathcal{U} = (B, V, *, <, >, \tilde{h})$ is a Kottwitz datum, with associated Shimura variety $\mathrm{Sh}(\mathbf{G}, \mathbf{X})$ (see [Gol12, §3.1]) such that the center of the simple \mathbf{Q} -algebra B is an imaginary quadratic field F . Let ℓ be a prime of good reduction for \mathcal{U} (see *loc. cit.* §3.3) and $\mathcal{K}^{(\ell)} \subset \mathbf{G}(\mathbf{A}_f^\ell)$ an open compact subgroup. Let $Sh_{\mathcal{K}^{(\ell)}, \ell}$ be the Kottwitz integral model of $Sh(\mathbf{G}, \mathbf{X})$ at level $\mathcal{K}^{(\ell)}$ (see *loc. cit.*

W. G. PRINCETON UNIVERSITY, FINE HALL, WASHINGTON ROAD, PRINCETON, NJ 08544-1000, USA

M.-H. N. INSTITUT DE MATHÉMATIQUES DE LUMINY, UNIVERSITÉ D'AIX-MARSEILLE, CAMPUS DE LUMINY, CASE 907, 13288 MARSEILLE CEDEX 9, FRANCE

E-mail address: wushijig@gmail.com, nicole@iml.univ-mrs.fr.

Date: February 26, 2013.

2010 *Mathematics Subject Classification.* Primary 14G35 ; Secondary 11F33, 11F55.

Key words and phrases. Hasse invariant, Galois representations, Shimura varieties, μ -ordinary locus.

§3.4]). Let $E = E(\mathbf{G}, \mathbf{X})$ be the reflex field of $\mathrm{Sh}(\mathbf{G}, \mathbf{X})$ and λ a prime of E lying above ℓ . Denote by $sh_{\mathcal{K}^{(\ell)}, \lambda}$ the special fiber of $Sh_{\mathcal{K}^{(\ell)}, \ell}$ at λ .

Theorem 1.1 (μ -ordinary Hasse invariant). *There exists an automorphic line bundle ${}^\mu\omega_{\mathcal{K}^{(\ell)}}$, and a section ${}^\mu H \in H^0(sh_{\mathcal{K}^{(\ell)}, \lambda}, {}^\mu\omega_{\mathcal{K}^{(\ell)}}^{\otimes(\ell^2-1)})$ such that:*

- (μ -Ha1) *The non-vanishing locus of ${}^\mu H$ is the μ -ordinary locus of $sh_{\mathcal{K}^{(\ell)}, \lambda}$.*
- (μ -Ha2) *There exists an integer $m \in \mathbf{N}$ such that $({}^\mu H)^m$ lifts to characteristic zero.*
- (μ -Ha3) *The construction of ${}^\mu H$ is compatible with varying the level $\mathcal{K}^{(\ell)}$.*

Corollary 1.2. *The μ -ordinary locus $sh_{\mathcal{K}^{(\ell)}, \lambda}^{\mu\text{-ord}, \min}$ in the minimal compactification $sh_{\mathcal{K}^{(\ell)}, \lambda}^{\min}$ is affine.*

Remark 1.3. We do not know the minimal value of m in (μ -Ha2). The Hasse invariant of Siegel varieties lifts i.e., $m = 1$, see [BN07].

1.2. Application. Let \mathcal{U} be as in §1.1. Suppose π is a cuspidal automorphic representation of $\mathbf{G}(\mathbf{A})$ with p -adic component π_p for every (rational) prime p . Given a (rational) prime ℓ , let $\mathcal{P}^{(\ell)}$ be the set of (rational) primes p different from ℓ such that π_p is unramified and \mathbf{G} is unramified at p . Let $\mathfrak{P}^{(\ell)}$ be the set of primes of F that are split and lie over some $p \in \mathcal{P}^{(\ell)}$.

Assume $\wp \in \mathfrak{P}^{(\ell)}$. One has a decomposition $\mathbf{G}(\mathbf{Q}_p) \cong \mathbf{Q}_p^\times \times \mathrm{GL}(n, F_\wp)$, where n is given by $n^2 = \dim_F \mathrm{End}_B V$. Write $\pi_p \cong \chi_p \otimes \pi_\wp$, with χ_p a character of \mathbf{Q}_p^\times and π_\wp a representation of $\mathrm{GL}(n, F_\wp)$.

Our result on Galois representations is:

Theorem 1.4. *Suppose π is a cuspidal automorphic representation of $\mathbf{G}(\mathbf{A})$ whose archimedean component π_∞ is an \mathbf{X} -holomorphic limit of discrete series representation of $\mathbf{G}(\mathbf{R})$ (see [Gol12, §2.3]). Assume ℓ is a prime (of \mathbf{Q}) of good reduction for \mathcal{U} . Then there exists a unique semisimple Galois representation*

$$(1) \quad R_{\ell, \iota}(\pi) : \mathrm{Gal}(\overline{F}/F) \longrightarrow \mathrm{GL}(n, \overline{\mathbf{Q}}_\ell)$$

satisfying the following two conditions:

- (Gal1) *If $p \in \mathcal{P}^{(\ell)}$ and \wp is a prime of F dividing p then $R_{\ell, \iota}(\pi)$ is unramified at \wp . In particular $R_{\ell, \iota}(\pi)$ is unramified at all but finitely many places.*
- (Gal2) *If $\wp \in \mathfrak{P}^{(\ell)}$ then there is an isomorphism of Weil-Deligne representations*

$$(2) \quad (R_{\ell, \iota}(\pi)|_{W_{F_\wp}})^{\mathrm{ss}} \cong \iota^{-1} \mathrm{rec}(\pi_\wp \otimes |\cdot|^{\frac{1-n}{2}}),$$

where W_{F_\wp} is the Weil group of F_\wp , the superscript ss denotes semi-simplification and rec is the Local Langlands Correspondence, normalized as in Harris-Taylor [HT01].

Remark 1.5. *Comparison with [Gol12, Th.1.2.1].* The improvement in Th. 1.4 with respect to *loc. cit.* is the removal of the assumption that some prime λ of E above ℓ is split in E .

2. CONSTRUCTION OF THE μ -ORDINARY HASSE INVARIANT

Assume henceforth, without loss of generality, that $a \leq b$. The assumption that ℓ is a prime of good reduction for \mathcal{U} implies that ℓ is unramified in E .

If λ is split in E , Th. 1.1 is well-known (see e.g., [Gol12, §4]). If $a = b$ then $E = \mathbf{Q}$, so λ is necessarily split in E . Hence we assume from now on that $a < b$ and that λ is inert in E .

As in [Gol12, §3.7], the Hodge bundle $\Omega_{\mathcal{K}^{(\ell)}}$ decomposes as

$$(3) \quad \Omega_{\mathcal{K}^{(\ell)}} = \Omega_{\mathcal{K}^{(\ell)},a}^{\oplus r} \oplus \Omega_{\mathcal{K}^{(\ell)},b}^{\oplus r},$$

where $\Omega_{\mathcal{K}^{(\ell)},a}$ (resp. $\Omega_{\mathcal{K}^{(\ell)},b}$) has rank a (resp. b) and r is the rank of B over F . Let $\omega_{\mathcal{K}^{(\ell)},a}$ (resp. $\omega_{\mathcal{K}^{(\ell)},b}$) be the determinant of $\Omega_{\mathcal{K}^{(\ell)},a}$ (resp. $\Omega_{\mathcal{K}^{(\ell)},b}$).

Let \mathcal{A} be an abelian scheme representing the universal isogeny class above $sh_{\mathcal{K}^{(\ell)},\lambda}$. As in (4.6) of *loc. cit.*, the Verschiebung $\text{Ver} : \mathcal{A}^{(\ell)} \rightarrow \mathcal{A}$ induces a map

$$(4) \quad \text{Ver}^* : \Omega_{\mathcal{K}^{(\ell)}} \rightarrow \Omega_{\mathcal{K}^{(\ell)}}^{(\ell)}.$$

Since λ is inert, the restrictions of Ver^* to $\Omega_{\mathcal{K}^{(\ell)},a}$ (resp. $\Omega_{\mathcal{K}^{(\ell)},b}$) have the form

$$(5) \quad \text{Ver}^*|_{\Omega_{\mathcal{K}^{(\ell)},a}} : \Omega_{\mathcal{K}^{(\ell)},a} \longrightarrow \Omega_{\mathcal{K}^{(\ell)},b}^{\ell} \text{ and } \text{Ver}^*|_{\Omega_{\mathcal{K}^{(\ell)},b}} : \Omega_{\mathcal{K}^{(\ell)},b} \longrightarrow \Omega_{\mathcal{K}^{(\ell)},a}^{\ell}.$$

Therefore, if $(\text{Ver}^*)^2$ denotes the composite of Ver^* with itself, then we have

$$(6) \quad (\text{Ver}^*)^2|_{\Omega_{\mathcal{K}^{(\ell)},a}} : \Omega_{\mathcal{K}^{(\ell)},a} \longrightarrow \Omega_{\mathcal{K}^{(\ell)},a}^{(\ell^2)}.$$

Let

$$(7) \quad {}^{\mu}h(\mathcal{A}) : \omega_{\mathcal{K}^{(\ell)},a} \longrightarrow \omega_{\mathcal{K}^{(\ell)},a}^{\otimes(\ell^2)},$$

be the top exterior power of that map, where we have used that $\omega_{\mathcal{K}^{(\ell)},a}^{(\ell^2)} = \omega_{\mathcal{K}^{(\ell)},a}^{\otimes(\ell^2)}$ since $\omega_{\mathcal{K}^{(\ell)},a}$ is a line bundle. The map ${}^{\mu}h(\mathcal{A})$ induces a global section

$$(8) \quad {}^{\mu}H(\mathcal{A}) \in H^0(sh_{\mathcal{K}^{(\ell)},\lambda}, \omega_{\mathcal{K}^{(\ell)},a}^{\otimes(\ell^2-1)}).$$

If \mathcal{B} is another representative of the universal isogeny class above $sh_{\mathcal{K}^{(\ell)},\lambda}$ and $\varphi : \mathcal{A} \rightarrow \mathcal{B}$ is an isogeny compatible with the endomorphism actions of \mathcal{A}, \mathcal{B} , then as in [Gol12, §4.2], the compatibility of Verschiebung with isogenies (Lemma 4.2.3 of *loc. cit.*) implies that $\varphi^*({}^{\mu}H(\mathcal{B})) = {}^{\mu}H(\mathcal{A})$. Hence we may omit reference to the representatives \mathcal{A} or \mathcal{B} and we have a section ${}^{\mu}H \in H^0(sh_{\mathcal{K}^{(\ell)},\lambda}, \omega_{\mathcal{K}^{(\ell)},a}^{\otimes(\ell^2-1)})$, which we call the μ -ordinary Hasse invariant.

3. PROOFS.

We begin with the proof of Th. 1.1. The following two lemmas and their corollaries will establish that ${}^{\mu}H$ satisfies $(\mu\text{-Ha1})$.

Lemma 3.1. *The Newton polygon \mathcal{N}^{ord} of the underlying isogeny class of abelian schemes of a μ -ordinary geometric point of $sh_{\mathcal{K}^{(\ell)},\lambda}$ has the following slopes:*

$$\left. \begin{array}{c} 0 \\ 1/2 \\ 1 \end{array} \right\} \text{ with multiplicity } \left\{ \begin{array}{c} 2ar \\ 2(b-a)r \\ 2ar \end{array} \right.$$

Proof. The case $r = 1$ follows from [Wed99, 2.3.2]. The case of general r follows subsequently from [Moo04, 1.3.1 and 3.2.9]. \square

Proposition 3.2. *The μ -ordinary locus is the maximal ℓ -rank stratum of $sh_{\mathcal{K}^{(\ell)},\lambda}$.*

Proof. The key point is that, by [RR96, Prop. 2.4(iv) and Th.4.2], the Newton polygon \mathcal{N}^{ord} described in Lemma 3.1 is the lowest among the Newton polygons of the underlying isogeny classes of abelian schemes corresponding to geometric points of $sh_{\mathcal{K}^{(\ell)}, \lambda}$. Let A be an abelian scheme with Newton polygon $\mathcal{N}(A)$. Then $\mathcal{N}(A)$ is symmetric and the ℓ -rank of A is the multiplicity of 0 (=the multiplicity of 1) as a slope of $\mathcal{N}(A)$. But if the multiplicity of 0 in $\mathcal{N}(A)$ is at least the multiplicity of 0 in \mathcal{N}^{ord} and $\mathcal{N}(A)$ lies on or above \mathcal{N}^{ord} , then by Lemma 3.1 we must have $\mathcal{N}(A) = \mathcal{N}^{\text{ord}}$. \square

Corollary 3.3. *The maximal ℓ -rank stratum of $sh_{\mathcal{K}^{(\ell)}, \lambda}$ has ℓ -rank $2ar$.*

Proof. This follows directly from Lemma 3.1 and the proof of Prop. 3.2. \square

Lemma 3.4. *Suppose A is an abelian scheme which is a representative of the underlying isogeny class of a geometric point of $sh_{\mathcal{K}, \lambda}$. Then ${}^\mu H(A) \neq 0$ if and only if the ℓ -rank of A is equal to $2ar$.*

Proof. One has $H^1(A, \mathcal{O}_A) \cong H^0(A, \Omega_A^1)$ and under this isomorphism the action of Frobenius on $H^1(A, \mathcal{O}_A)$ corresponds to that of Verschiebung on $H^0(A, \Omega_A^1)$. Hence [Mum08, §15] implies that the ℓ -rank of A equals the semisimple rank of $(\text{Ver}^*)^j : \Omega \rightarrow \Omega^{(\ell^j)}$ for all $j \in \mathbf{N}$. Since $\dim A = (a+b)r$, keeping in mind (3) and using the last corollary of §14 of *loc. cit.*, $(\text{Ver}^*)^j$ is semisimple for $j \geq a+b$. Therefore the ℓ -rank of A equals the rank of $(\text{Ver}^*)^j$ for $j \geq a+b$. We take the $\otimes(a+b)$ iterate of the section ${}^\mu H(A)$, see (7). It is clear that ${}^\mu H(A) \neq 0$ if and only if ${}^\mu H(A)^n \neq 0$ for any $n \in \mathbf{N}_{>0}$, in particular for $n = a+b$.

Since $a \leq b$, both $\text{Ver}^*_{|\Omega_{\mathcal{K}^{(\ell)}, a}}$ and $\text{Ver}^*_{|\Omega_{\mathcal{K}^{(\ell)}, b}}$ have rank at most a . So also $(\text{Ver}^*)^j_{|\Omega_{\mathcal{K}^{(\ell)}, a}}$ and $(\text{Ver}^*)^j_{|\Omega_{\mathcal{K}^{(\ell)}, b}}$ each have rank at most a . By (3), $(\text{Ver}^*)^j$ has rank at most $2ar$.

The ℓ -rank of A equals $2ar$ if and only if the rank of $(\text{Ver}^*)^j$ is $2ar$ for $j \geq a+b$. In turn, the rank of $(\text{Ver}^*)^j$ is $2ar$ if and only if both $\text{Ver}^*_{|\Omega_{\mathcal{K}^{(\ell)}, a}}$ and $\text{Ver}^*_{|\Omega_{\mathcal{K}^{(\ell)}, b}}$ have rank a . Since $\Omega_{\mathcal{K}^{(\ell)}, a}$ and $\Omega_{\mathcal{K}^{(\ell)}, a}^{(\ell^{2(a+b)})}$ are rank a vector bundles, the determinant of a map between them is nonzero if and only if it has rank a . \square

We now conclude the proof of Th. 1.1.

Proof of Th. 1.1: Combining Prop. 3.2, Cor. 3.3 and Lemma 3.4 gives $(\mu\text{-Ha1})$. By [LS12b, Prop. 7.14] (or [LS12a] in the compact case), there exists $k \in \mathbf{N}$ such that $\omega_{\mathcal{K}^{(\ell)}, a}^{\otimes(k(\ell^2-1))}$ extends to an ample line bundle on the minimal compactification $sh_{\mathcal{K}^{(\ell)}, \lambda}^{\min}$. Given this ampleness result, the existence of a lift of some power of ${}^\mu H$ follows by a well-known cohomological argument coupled with the Koecher principle (cf. [Gol12, Lemma 4.4.1]). Thus $(\mu\text{-Ha2})$ is established. Finally $(\mu\text{-Ha3})$ is proved in the same way as Th.4.2.4 of *loc. cit.* \square

Next we note that Cor. 1.2 is an immediate consequence of Th. 1.1:

Proof of Cor. 1.2: The nonvanishing locus of a section of an ample line bundle on a projective scheme is affine. \square

Finally we note how Th. 1.4 follows from Th. 1.1.

Proof of Th. 1.4: The proof is analogous to the proof of [Gol12, Th.1.2]: Let $\mathcal{K} \subset \mathbf{G}(\mathbf{A}_f)$ be an open compact subgroup such that $\mathcal{K} = \mathcal{K}^{(\ell)} \mathcal{K}_{(\ell)}$ with $\mathcal{K}^{(\ell)} \subset \mathbf{G}(\mathbf{A}_f^\ell)$ and $\mathcal{K}_{(\ell)} \subset \mathbf{G}(\mathbf{Z}_\ell)$. Let $Sh_{\mathcal{K},E}$ be the model of $\mathrm{Sh}(\mathbf{G}, \mathbf{X})$ at level \mathcal{K} over E as defined in §3.2 of *loc. cit.*. Let $Sh_{\mathcal{K},\ell}$ be the normalization of $Sh_{\mathcal{K}^{(\ell)},\ell}$ in $Sh_{\mathcal{K},E}$. Let $\pi : Sh_{\mathcal{K},\ell} \rightarrow Sh_{\mathcal{K}^{(\ell)},\ell}$ be the natural projection.

Using $(\mu\text{-Ha2})$, let ${}^\mu H_{\mathcal{K}^{(\ell)}}^{\mathrm{lift}}$ be a lift of a power ${}^\mu H$ and let ${}^\mu H_{\mathcal{K}}^{\mathrm{lift}}$ be the pullback of ${}^\mu H_{\mathcal{K}^{(\ell)}}^{\mathrm{lift}}$ to $Sh_{\mathcal{K},\ell}$ along the projection π .

Let $\mathcal{H} = \mathcal{H}_{\mathcal{P}^{(\ell)}}(\mathbf{G}, \mathbf{Z}_\ell)$ be the spherical Hecke algebra of \mathbf{G} with values in \mathbf{Z}_ℓ , trivial at places outside $\mathcal{P}^{(\ell)}$ (see §6.1 of *loc. cit.* for a more detailed definition).

Theorem 3.5. *Suppose \mathcal{V} is an automorphic vector bundle on $Sh_{\mathcal{K},\ell}$ and $f \in H^0(Sh_{\mathcal{K},\ell}, \mathcal{V})$ is nonzero modulo λ . Then for all $j \in \mathbf{N}$, the product $({}^\mu H_{\mathcal{K}}^{\mathrm{lift}})^{\ell^j} f \in H^0(Sh_{\mathcal{K},\ell}, \omega_{\mathcal{K},a}^{\otimes(\ell^2-1)\ell^j} \otimes \mathcal{V})$ is nonzero modulo λ and satisfies*

$$(9) \quad T(({}^\mu H_{\mathcal{K}}^{\mathrm{lift}})^{\ell^j} f) \equiv ({}^\mu H_{\mathcal{K}}^{\mathrm{lift}})^{\ell^j} T(f) \pmod{\lambda^{j+1}} \text{ for all } T \in \mathcal{H}.$$

Proof. Since the μ -ordinary locus is dense [Wed99, Th.1.6.2], the product $({}^\mu H_{\mathcal{K}}^{\mathrm{lift}})^{\ell^j} f$ is nonzero modulo λ by $(\mu\text{-Ha1})$. As in the proof of [Gol12, Th.6.2.1], $(\mu\text{-Ha3})$ implies (9). \square

Given Th. 3.5, the remainder of the argument to establish Th. 1.4 is identical to §§6.3–6.4 of *loc. cit.* \square

Remark 3.6. A tremendous advantage of our μ -ordinary Hasse invariant is that it satisfies all key properties of the classical invariant. Its applications will thus follow the classical blueprint: to Galois representations (as we illustrated briefly above), but also immediately the (non-effective) existence of its canonical subgroup thanks to the elementary [Far11, Prop.3], and thus also applications to explicit constructions of eigenvarieties, etc.

4. ACKNOWLEDGMENTS

We thank the Max-Planck-Institut für Mathematik for a year-long membership in 2011 (M.-H. N.) and also for a short visit in May 2011 (W.G.). In particular, the natural albeit key idea of considering higher powers of Verschiebung occurred to M.-H. N. on his very first Monday at MPIM.

W.G. thanks Pierre Deligne, Richard Taylor, Barry Mazur, Elena Mantovan, David Geraghty, Benoît Stroh, Jacques Tilouine and Vincent Pilloni for helpful conversations. W. G. is happy to acknowledge support from 10 BLAN 114 01 ANR ARSHIFO, a Simons Travel Grant and NSF MSPRF.

REFERENCES

- [BN07] S. Böcherer and S. Nagaoka, *On mod p properties of siegel modular forms*, Math. Ann. **338** (2007), 421–433.
- [DS74] P. Deligne and J.-P. Serre, *Formes modulaires de poids 1*, Ann. Sci. ENS **7** (1974), no. 4, 507–530.
- [Far11] L. Fargues, *La filtration canonique des points de torsion des groupes p -divisibles*, A.S.E.N.S. **44** (2011), 905–961.
- [Gol12] W. Goldring, *Galois representations associated to holomorphic limits of discrete series I: Unitary groups*, May 2012, With an appendix by. S.-W. Shin. Submitted, available at www.math.harvard.edu/~wushi.

- [HT01] M. Harris and R. Taylor, *The geometry and cohomology of some simple Shimura varieties*, Annals of Math. Studies, vol. 151, Princeton Univ. Press, 2001.
- [LS12a] K.-W. Lan and J. Suh, *Vanishing theorems for torsion automorphic sheaves on compact PEL-type Shimura varieties*, Duke Math. J. **161** (2012), no. 6, 1113–1170.
- [LS12b] ———, *Vanishing theorems for torsion automorphic sheaves on general PEL-type Shimura varieties*, 2012, Preprint, available at <http://www.math.umn.edu/~kwlan>.
- [Moo04] B. Moonen, *Serre-Tate theory for moduli spaces of PEL-type*, Ann. Sci. ENS **37** (2004), no. 2, 223–269.
- [Mum08] D. Mumford, *Abelian varieties*, Tata Inst. of Fund. Res. Studies in Math., vol. 5, Hindustan Book Agency, New Delhi, India, 2008, With appendices by C. P. Ramanujam and Y. Manin. Corrected reprint of the second (1974) edition.
- [RR96] M. Rapoport and M. Richartz, *On the classification and specialization of F -isocrystals with additional structure*, Comp. Math. **103** (1996), 153–181.
- [Tay91] R. Taylor, *Galois representations associated to Siegel modular forms of low weight*, Duke Math. J. **63** (1991), no. 2, 281–332.
- [Wed99] T. Wedhorn, *Ordinariness in good reductions of Shimura varieties of PEL-type*, Ann. Sci. ENS **32** (1999), no. 5, 575–618.